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Effects of schematising on mathematical development

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The purpose of this study was to investigate whether introducing schematising to children in early childhood (ages five to six) promotes better learning outcomes in later mathematics (age seven). This was done using a longitudinal, quantitative study with a quasi-experimental design. With the help of teachers and a teacher-trainer, pupils in the experimental group of our research study were exposed to, and participated actively in, schematising activities over the course of a year during their early childhood. In grade two (ages five to six) at three Dutch primary schools, children practised schematising emerging from play activities. In the control group, children were not 'trained' nor did they participate in schematising activities. We hypothesised that the experimental group would demonstrate an enhanced ability in schematising and mathematics.

In this article, the research outcomes of our study on the differences in the performance of schematising and mathematical development between both groups are presented. We conclude that pupils in the experimental group demonstrated significantly better results on schematising and mathematics.

Le but de cette étude était d'étudier si la présentation de schémas aux enfants de 5 à 6 ans favorisait des meilleurs résultats en mathématiques à l'age de 7 ans. Pour atteindre leur objectif, les chercheurs ont lancé une étude longitudinale et quantitative, en suivant une méthodologie quasi-expérimentale. Avec l'aide des professeurs et d'un professeur-formateur, les élèves dans le groupe expérimental de l'étude ont participé activement dans la pratique de schémas (transformation des schémas de la pensée) pendant toute une année dans la période préscolaire. Dans la grade 2 (âges 5 à 6 ans), dans trois écoles primaires au Pays-Bas, les enfants ont pratiqué des schémas, liées aux activités de jeu. Dans le groupe de contrôle, les enfants n'étaient pas 'formé' à cet exercice, et n'ont pas participé dans des activité de schémas. Nous avons avancé l'hypothèse que le groupe expérimental démontrerait une capacité augmentée pour utiliser des schémas, surtout en mathématiques.

Dans cet article, nous présentons les résultats de notre recherche sur les différences dans l'exécution et le développement de schémas entre les deux groupes. Nous concluons que les élèves dans le groupe expérimental ont sensiblement amélioré leurs capacités, surtout en mathématiques.

Zweck der Studie war zu untersuchen, ob die Einführung von Schematisierung bei Kindern im frühen Kindesalter (5 bis 6 Jahre) später (im Alter von 7 Jahren) bessere Lernergebnisse in der

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Mathematik bewirkt. Dies geschah unter Verwendung eines längsschnittlichen quantitativen und quasi-experimentellen Untersuchungsdesigns. Mit Hilfe von Lehrkräften und eines Lehrertrainers nahmen Schulkinder in der Experimentalgruppe über ein Jahr hinweg an Aktivitäten von Schematisierung teil. In der zweiten Klasse (im Alter von 5 bis 6) in drei holländischen Grundschulen übten Kinder das Schematisieren aus Spielaktivitäten heraus. In der Kontrollgruppe wurden die Kinder weder 'trainiert' noch nahmen sie an Schematisierungsaktivitäten teil. Wir nahmen an, dass die Experimentalgruppe eine erhöhte Fertigkeit im Schematisieren und in Mathematik.

Die Ergebnisse aus dieser Forschung werden berichtet. Wir kommen zu dem Schluss, dass Schulkinder in der Experimentalgruppe signifikant bessere Resultate beim Schematisieren und in Mathematik zeigten.

El propósito de este estudio era investigar si introduciendo esquematizaciones (schematising) a los niños en su temprana infancia (cinco a seis años de edad) promueve un mejor aprendizaje posterior de matemáticas (siete años de edad). Esto fue realizado haciendo uso de un estudio longitudinal, cuantitativo con en diseño casi-experimental. Con la ayuda de profesores y estudiantes a profesores, los ninos en el grupo experimental de nuestro estudio fueron expuestos a, y participaron activamente en, actividades esquematizadoras (schematising) durante un año en su primera infancia. En el segundo grado (edad de cinco a seis años) en tres escuelas primarias holandesas, los niños practicaron esquematizaciones (schematising) emergiendo de sus actividades de juegos. En el grupo de control, los ninos no fueron 'entrenados' ni participaron en actividades esquematizadoras (schematising). Nuestra hipótesis era que el grupo experimental demostraría un mayor habilidad para esquematizaciones (schematising) y matemáticas.

En este articulo presentamos los resultados de nuestro estudio acerca de las diferencias de desempeño en cuanto a esquematizaciones (schematising) y matemáticas desarrollada entre ambos grupos. Concluimos que los alumnos del grupo experimental demostraron significativamente mejores resultados en squematizaciones (schematising) y matemáticas.

Keywords: Holland; mathematics; play activities; schematising

Theoretical framework

The theory and research described in this article are based on the Vygotskian approach to human development. From this perspective we studied early schematising activity of young children, and tried to promote the development of schematising in these children. Conceptually, we conceive of schematising as a special (structured) form of symbolisation. Research (such as that of Deloache *et al.*, 1998) demonstrated that symbols do not have a self-evident meaning for children. The same is true for schematisations. Nevertheless, as we have demonstrated in previous case studies (e.g. van Oers, 1994), schematising appears to be an accessible activity for young children (at least from the age of 5), if it is introduced in a functional way in children's play activities.

Concept of schematising

The concept of schematising and schematisations in young children's mathematics education is receiving more attention than ever before (see Carruthers & Worthington, 2003). There appears to be a gap between the concrete practical reasoning of young children and the logical-symbolic reasoning expected in later mathematical development. Investigating the possible benefits of schematising seems to be an important step towards bridging this gap (Dijk *et al.*, 2004).

A schematisation can be described as a graphic representation of reality, by which one can make statements about that reality. By means of representations such as schemes, people can organise their knowledge and thoughts. In this study, a schematising activity is considered to be every cognitive activity aimed at the construction and the improvement of symbolic representations of an element of physical and sociocultural reality. An example of this is as follows: Imagine a child is singing a song. Every syllable of every word in a song has a certain pitch, sound and accent. One syllable has to be sung very loud, very low and very long, whereas another syllable has to be sung a little quieter, higher and shorter. The teacher can ask the child to picture the melody and, in doing so, the teacher asks the child to schematise the song. In fact, what the teacher asks the child to do is to represent the child's reality symbolically. This requires the child to think about the function of symbols and schemes and the relationship between the symbols and the song. By making a staff (which is basically a scheme), the child can organise the sounds in the song. Additionally, by means of symbols in the staff, the child can represent the way the syllable, and the song, has to be sung.

This type of a schematisation is a dynamic schematisation because of the changes this scheme represents (see, for example, Figure 1).

Static schematisations do not represent changes, transformations or translations. They only describe a status quo or a state of equilibrium (see, for example, Figure 2).

Dynamic schematisations require more extensive understanding in comparison to static schematisations. Dynamic schematisation presumes a higher level of understanding of relationships between states. This understanding is necessary to create representations of action, activities of change, transformation, and so on. Most mathematical activities are based on the use and construction of such dynamic schematisations (van Oers, 2002; Carruthers & Worthington, 2003). In order to let

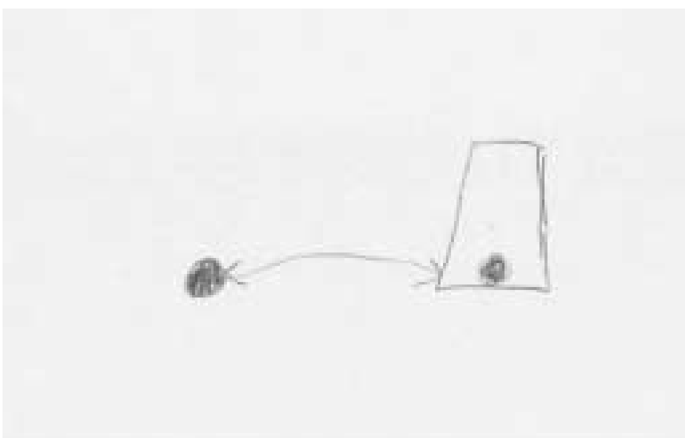

Figure 1. Dynamic schematisation

Figure 2. Static schematisation

the differences between static and dynamic representations be clear, we present Table 1.

Schematising and mathematics

Mathematical reasoning requires mathematising. Mathematising is, according to Freudenthal (1973), the ability to organise one's own field of experiences. In mathematics education, children are often asked to organise quantitative or spatial data in

Table 1. Differences between static and dynamic schematisation

Dimension	Static schematisation	Dynamic schematisation
Transformation	Similarity between image and object	Drawings representing action; change, processes, movement, transformation
	Use of words and letters referring to individual objects	Drawings representing a line of thought/reasoning of other process symbols
	Being able to explain or carry out the relationship between drawing and object, sign and meaning: equivalence	Being able to explain or carry out the relationship between drawing and object, sign and meaning: transformation
Association	One symbol per counted object	Drawings representing movement or modifications and the objects involved
Narrative structure	Narratives as descriptions of states	Narratives with a 'plot'
Use of operators	Use of numerals and symbols referring to individual objects	Making use of operators as productive symbols
Meaningfulness	Mechanically associating symbols (reproducing, rote learning)	Synthesising in a meaningful way different meanings associated with the sign (relating, arguing)

order to solve mathematical problems. Moreover, children are often required to interpret symbols in order to organise the data. This means that children have to reorganise, translate or transform functionally related data into new forms or configurations. Consequently, a mathematical activity, or mathematising, is basically a *dynamic* activity. The dynamic aspect of such activities refers to the transformation and translation of connections, functions, symbols and variables. Children are required to transform data or thoughts into symbols and then translate symbols back into data or statements. This process is very difficult for young children because, in early childhood, children tend to lack familiarity with organising and structuring data using mathematical reasoning and symbols. According to Cobb *et al.* (1997), 'the struggle for mathematical meaning can be seen in large part as a struggle for means of symbolising' (p. 161). In our view, this struggle for meaning occurs schematically in mathematising.

Dynamic schematising may prove to be an effective strategy to facilitate mathematical reasoning given that such schematising activities are aimed at the construction and improvement of symbolic representations of changes or transformations in a child's reality. If children are confronted with a mathematical task, they have to interpret the task: What is it about, what does it ask for, what is the purpose of the task, and, lastly, what does this mean to me? To solve the task, children have to invent ways to not only structure the steps required to complete the task but also to determine the meaning of the symbols being used. And in order to make these mental structures into objects for reflection and interpersonal communication, it is necessary to make symbolic representations—that is, schemes of them. Schematisations are a good way to represent and structure thoughts as they allow people to show connections, transformations and translations of the initial data. By clarifying the steps in their line of thought, people can reflect on their own thinking process and inform other people about it. The practice of creating graphic representations of thoughts, and lines of thought and the use of one's own notational systems, is expected to improve mathematical reasoning since these strategies can help children understand the representational function of representations such as mathematical symbols, schemes and relations. If children understand the function of symbols, they can develop ways to solve problems. This means that, if they are given a mathematical task, they will know how to organise the information provided and how to translate or transform it into a meaningful solution.

Schematising in early childhood

Many children have difficulties with mathematical reasoning. These problems usually manifest themselves as soon as children reach grade three (age six to seven) in primary school because, at that point, there is a qualitative change in children's activities and even in the way they are taught and how they learn. In early childhood, children do not necessarily need to organise their knowledge and thoughts in the way they organise them in later development. According to El' konin (1972), as children age, different interests and capacities emerge. He theorises that development can be classified into five periods spread over the years from birth to sixteen years of age. At different stages in a child's development, a child relates differently to his or her environment. Additionally, he contends that specific capabilities and interests are characteristic for a child in a certain stage. A stage is defined as a specific collection of functions such as thinking, memory, emotions and language. As a result of this collection of functions, each child has his or her own characteristic relationship with the cultural environment. Within these stages, certain tensions exist because, at a certain point, a child becomes ready to develop new skills which are not characteristic of the stage of development he or she is in at that moment. The tensions result from the desire to use the skills that are characteristic of the next stage of development. In essence, there are tensions between what a child wants to do and what he or she is able to do on his or her own. The characteristic form that a child uses to interact with his or her environment at a certain stage in his or her development is what is called the 'leading activity'.

For children between the ages of three and seven, the leading activity is play. When they reach approximately seven years of age, a discrepancy arises between the things children want to do and the things they can do. This discrepancy can only be solved by introducing elements from the next stage of development. 'This crisis is the psychological motor of development' (van Oers & Wardekker, 1997, p. 193). After *play* as the leading activity, at around seven years of age, *learning* develops into the leading activity. In this period, a foundation for later constructive learning is formed. This is a type of directed learning that is based on the use of models and schemes and on discussions regarding their meaning. In this phase, children are motivated enough to get involved in a form of 'learning to know' (van Oers & Wardekker, 1997) and also conceptualise concrete reality in terms of abstract models. In the words of Davydov (quoted in van Oers & van Dijk, 2004), 'In this period children are trying to ascend from the abstract to the concrete.' According to Davydov, symbolic models are the best means of moving from the abstract to the concrete and the other way around.

According to the theory of leading activities, finding strategies for promoting new learning processes in early childhood education that can promote the emergence of the next leading activity is imperative (van Oers, 1994). The roots of learning processes that will play a leading role in later development are found in the context of play. In play activities, children also learn to communicate and coordinate their own activities. In this phase, children can learn to deal with schematical representations of an element of reality, such as schemes, diagrams, drawings or symbols. In this period, it is possible to lay a foundation for later conscious, constructive learning (van Oers & Wardekker, 1997, pp. 192–193)—that is, learning activity. The acquisition of strategies to incorporate schematic representations into mathematical reasoning is presumably an important element of this foundation.

Vygotsky emphasised the fact that a teacher should build on a child's own interests and capabilities as a starting point for further development and, in doing so, try to convert these elements into a new form. Therefore, educational instruction should slightly exceed a child's actual development (see Vygotsky, 1978). It should offer children the tools needed to create new and familiar ways of thinking and communicating. This so-called developmental education attempts to stimulate a child's development by enriching their activities and by starting from their own capabilities, especially with regard to their ability to use symbols and language. To enhance their symbolic capabilities, we can encourage children to invent graphic representations to communicate their thoughts and ideas. As Carruthers and Worthington (2003) state, 'children's own mathematical graphics supports children in developing their competences' (p. 78). Early childhood education should therefore assist children in improving these schematisations (including notations and schemes). The use of schematical representations could be a very important strategy for improving this process. As early as the *play stage of development,* we should attempt to enrich their play activity with schematisations. This can provide children with a rough understanding of the function of symbols and schemes so that they can be used when children encounter formal mathematical symbols in the later stages of mathematical reasoning. Consequently, early childhood education can be improved by introducing the use of schematical representations.

Why learn to schematise?

Making symbolical representations is an important feature of mathematics education, since mathematics is essentially based on symbol use. By encouraging young children to make their own representations, they can learn to recognise the relevance of symbols. Using their self-made representations, children can learn with the help of others how to reflect on what they have done and what they were meant to do. They therefore learn to represent relationships between objects (things, numbers, variables, etc.) using schematisations. These schematisations give structure to their thoughts and provide a means for the child to communicate mathematical reasoning.

In early childhood, children rarely use written symbols, schemes, and so on to represent their thoughts. However, as they grow older, they are required to use this form of notation more frequently, especially with respect to mathematics activities. This notating is what Pimm (1987) calls 'recording'. Pimm recognises children's problems in mathematics education and mathematical reasoning. He wrote, 'Pupils frequently fail to have a clear idea of why they are recording and, without any feeling for the purpose, it is difficult to discover what, for example, is ambiguous or insufficient in some way' (Pimm, 1987, p. 137). Children often do not understand why they have to use this mathematical language full of difficult symbols and imperceptible relationships. They are thus unaware of the multiple functions of symbols. According to Pimm (1987), we can attribute at least two main functions to symbols:

- Communication: through symbols we can communicate;
- Thinking device: symbols support 'problem-solving'. By reinterpreting a symbol, or by notating it in another more familiar way, we can make the problem recognisable.

Evidently, we are able to reflect through the use of symbols. 'It is largely by the use of symbols that we achieve voluntary control over our thoughts' (Skemp, cited in Ruckstuhl, 2001, p. 15). It is desirable that children learn to manage their thoughts in later development (from the age of seven onward), but this is not an easy process. Children in early childhood are familiar with working with visible objects and relationships. However, in mathematical education, they have to work with invisible objects and relationships. These objects and relationships have to be represented by symbols. Thus, pupils need to learn to work with representations using symbols and schemes. Pape and Tchoshanov (2001) propose that representations are an important feature of mathematics. Consequently, according to them, in mathematics education, representations must be thought of as tools for reasoning, explaining and justifying. As educators, we must teach children the function of these tools. It is necessary to develop children's understanding (Abrantes, 2002) by introducing tools to help them in this process of development. Self-invented schemes built on self-invented notational systems are the tools we want to initiate. When children are given the opportunity to invent their own strategies, schemes or notations to solve problems and to make their own representations of mathematical problems, they presumably will be more accepting of conventional symbols when they are introduced later in mathematics education (Dijk *et al.*, 2004; Munn, 2006). By allowing children to express their thoughts in their own ways to begin with, we use already existing symbols cemented to their conceptual structure (Skemp, 1989, p. 103).

Every child who enters elementary school has already developed his or her own range of skills, including mathematical skills (McPherson & Payne, 1997). At school, the teacher is expected to teach children a new kind of thinking—namely, mathematical thinking. We conceive of mathematics as 'an activity of systematically organising a concrete or mental domain in terms of quantitative and/or spatial relationships, constructing methods for problem solving related to that activity, as well as finding good reasons for this method' (van Oers, 1996, p. 75). Unfortunately, in traditional mathematics education, the necessary symbols are not introduced to children in a way that helps them see the purpose of using them. If children do not know why they have to solve mathematical sums, and there is no clear reason for them to participate in a mathematical activity and solve problems, children will not succeed in using the conventional operator signs used in mathematics education, nor will they understand them. When children fail to understand the reason for written methods, but are forced to use them at school, a gap arises between this formal knowledge and children's informal knowledge (Ginsburg, 1977; Hughes, 1986).

Introduction of schematising through educational strategies

Although van Oers (1994, 1996) has demonstrated that schematising activities are accessible to young children, it is not clear if this can be improved by education. Venger (1986) found evidence to support the theory that schematising can be taught. However, in his study, control groups were not used. As a result, we initially could not exclude the possibility that schematising is a product of general cognitive development. In order to determine if learning to schematise impacts mathematical development, we endeavoured to find a group of children that distinguished itself positively from another group on the ability of schematising. As we did not expect to find such a group, we invested a year in creating experimental groups which were likely to perform significantly better on schematising. In essence, we attempted to teach this in early childhood education in the context of play activities (for further explanations, see section on 'Intervention'). Consequently, we would be able to determine if learning to schematise could promote mathematical thinking.

Methods

Research question, hypothesis and design

The main research questions investigated by our research project were:

(1) Can schematising be learned?
(2) When compared with the culture of the control group, are schematising activities of more quality and a more integral aspect of the classroom culture in the experimental group?
(3) Are there differences in the mathematical learning outcomes between both groups (experimental or control)?

Our overall hypothesis was that if children are provided assistance with respect to finding an appropriate means of dealing with mathematical problems, and if they are given the relevant tools, they will achieve greater success in realistic mathematical activities and will also have better learning outcomes in later development.

To test this hypothesis, a quasi-experimental study was conducted with a control and an experimental group. In the experimental group, a teacher-trainer assisted teachers and pupils with schematising. In the context of the children's play, special attention was given to both the schematisation of dynamic processes and reflection on the schematisations. The teacher (with the support of the teacher-trainer) encouraged the children to make structured drawings of the situation, processes or activities. Care was taken that the drawings were functional for the activities of the children. In the control group, no special attention was given to dynamic schematisation and reflection during mathematical activities. This group just carried out its regular daily classroom practice.

We used the design-based research approach for our research project. A design experiment is characterised by the interaction between thought experiments and teaching experiments (see, for example, Bakker *et al.*, 2003; Cobb *et al.*, 2003). This means that, throughout the intervention process, we developed, in cooperation with teachers, the contents and course of the schematising activities. The researcher's proposals are tried out in practice, changed if necessary and theoretically reflected by the researcher. This led to a new theory-based proposal concerning new schematising activities in the children's thematic play. Thus, the design research is basically an iterative process between theory and practice. This is a theory-driven process, that (in our case) was informed by a Vygotskian approach to education.

Participants

Three schools, all using a Vygotsky-based approach to education, introduced schematising in their early childhood education (five-year-olds) for one school year. The experimental group was comprised of 75 pupils. Three other schools, also committed to the developmental education approach, functioned as the control group and therefore did not introduce schematising in early childhood education. This group was comprised of 58 pupils.

The schools were selected on the basis of previous contacts, their willingness to participate in the experiment, their approach to education and their contact with a teacher-trainer involved in our project. The six schools were divided among three research pairs (every control school was matched to an experimental school). The pairs were matched according to the amount of time a teacher had worked using this Vygotskian view of education, their student population, the number of students participating, and their location (urban or rural). All schools that participated were public schools. Two of the schools were situated in a large city (Amsterdam), two of them were located in medium- to large-sized cities and the remainder were situated in villages close to one of the larger cities. Two of the schools (one in the experimental group and one in the control group) were comprised predominantly of children from a non-Dutch background. As a result, the descriptions provided here cannot be generalised to all Dutch schools. However, we considered the comparison between the two groups to be sufficiently valid given the similarity between the two groups with respect to the school population, the degree to which teachers were prepared for this project, the use of curriculum materials and the length of time the teachers had worked with developmental education.

Study design

In September, 2002 we administered a pre-test in order to determine children's understanding of elementary mathematical skills and their understanding of numbers. In October, 2002, our intervention period started. Until June, 2003, a teacher-trainer assisted the experimental condition in schematising. In Table 2, the comparison between both conditions is presented.

In June, at the end of the intervention period, the 'schematising test' was administered in order to determine if schematising could be learned. At the start of grade

Table 2. Comparison between experimental and control setting

Experimental condition	Control condition
3 schools	3 schools
7 classes	6 classes
9 teachers	7 teachers
N=75	*N*=58
Schematising systematic	Schematising incidentally
Schematising complete	Schematising incomplete
Schematising static and dynamic	Schematising static (if even)
Attention for reflection on schemes	No attention for reflection
Specific role teacher-trainer	No specific role teacher-trainer

three, in September, 2003, the post-intervention period started. No attention was paid to schematising in both conditions from this moment on. In February, 2004, we administered our post-test and determined if our intervention had been successful.

Intervention

During the intervention period, the experimental groups participated in schematising activities. This intensive application was aimed at static and dynamic schematisation and at frequent, systematic and reflective schematising. To illustrate this, we provide an example of how a schematising activity was introduced: A child builds a castle. He asks his teacher how he could show his dad, who never visits school, his castle and how he built it. Using guiding questions, the teacher suggests ways he could show his father what he did. The child realises that he could make a drawing of his castle. The teacher then helps the child to transform the drawing into a schematic form. She shows the child how to count the blocks and how to graphically represent the same number of blocks in the scheme. The teacher-trainer assisted the teacher in using guiding questions and developing the schematising activity. This schematising activity emerged from a child's meaningful play activity. Throughout the experimental year, the teachers, teacher-trainer and researcher endeavoured to introduce schematising always in meaningful contexts. These endeavours were video-taped. In the post-intervention period, regular mathematics education was taught to both groups without emphasising schematising activities. After all, we sought to determine the extent to which early schematising influenced mathematical learning in grade three. In this grade, children are first confronted with mathematics instruction and begin mathematical problem-solving.

In our study, teachers required help from the teacher-trainer in order to learn how they could enrich children's activities. Like every worthwhile innovation, this took time. Nonetheless, we contend that the design research approach was the best approach for our experimental intervention process for two reasons: 1) we were able to ensure that the experimental innovation became meaningful for the teachers; and 2) we were able to take the complexities of schematising activities with the pupils into account.

The teacher-trainer visited the three experimental schools several times in the experimental year of our research project. He also visited the schools during the first year of our research project when the children were in grade two (age five to six). After each visit, the researcher and the teacher-trainer reflected on their experiences in the classroom. After this reflection, they were able to improve upon the experimental design. Both the researcher and the teacher-trainer observed several activities the children participated in and looked for ways to transform these activities with the help of schematising. Evidently, the people involved learned by doing. The teacher-trainer taught the teacher and pupils to create relevant schematising activities based on play activities. He also taught them how to participate in schematising activities. When children were participating in activities, he encouraged the teacher to transform the activities into schematising activities.

Our teacher-trainer also visited the control schools. However, there was a marked difference in his interventions in the control group classrooms as compared with the experimental group classrooms. Unlike the experimental groups, where the teacher-trainer placed emphasis on dynamic schematisation and reflection, no such activity was conducted with the control group. In short, while the teacher-trainer attempted to make the learning process of schematising as complete as possible in the experimental group, this was omitted with the control group. Here, the teacher-trainer only assisted the teacher in a general way with implementing the concept of developmental education (education based on the Vygotskian theory, as explained in the theoretical framework).

Instruments

Number sense (UGT, Test Moment 1). We began the research project in September, 2002 using a longitudinal pre-test/ post-test design with an experimental group and a control group. In order to determine the children's understanding of elementary mathematical skills and their understanding of numbers, the Utrechts Getalbegrip test (UGT) (Van Luit *et al.*, 1998) was administered. This test served as a pre-test to determine if there were differences in prerequisite mathematical understanding. It is one of the very few tests that can be administered in early childhood. The test does not measure other mathematical skills (such as working with symbols and mathematical problem-solving), since these abilities are taught in grade three and not earlier.

The schematising test (Test Moment 2). At the end of grade two, we administered the researcher-created 'schematising test' that was intended to measure how schematising activities proceeded at the end of our intervention period and if we had succeeded in teaching the experimental group schematising skills. One of the researchers videotaped the 'schematising tests'. In this test, the children were asked, one by one, to solve three schematisation problems. The questions were as follows:

Question 1:

The researcher showed the child a little pot with a red marble in it. Next to the pot, lay a green marble. The researcher asked the child to watch and see what happens. The researcher then took the red marble out of the pot and put the green one in the pot. Then he asked the child to draw what he or she saw.

The resulting graphical representations were expected to resemble the illustrations shown in Figure 3.

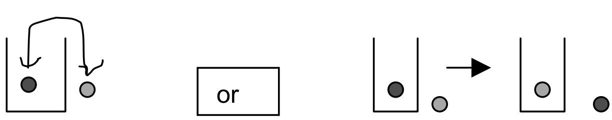

Figure 3. Answer to Question 1 of the 'schematising test'

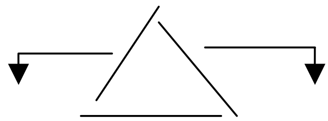

Figure 4. Question 2 in schematising

Question 2:

The researcher explained that, in this drawing, the child would be able to see a triangle made of three little bars. Then the child was given three bars. Following this, the child was asked to look at the drawing and show the researcher what should be done with the bars.
The child's drawing was expected to looked like that illustrated in Figure 4.
The child should have noted the purpose of the two arrows, which illustrate that the two sides of the triangle should be moved to the bottom of the diagram.

Question 3:

The researcher told the child about a little mouse which had been walking around the classroom. The mouse had been following the route illustrated by the picture below. The researcher asked the child to describe the route the mouse took and asks if the child can walk the same route in the classroom.

Collectivity: the schematisation score

The classrooms in our study were from schools that were committed to a Vygotskian approach to education, and as such the pupils were used to working in meaningful activity settings, with collaborative learning, reflective activities, and teacher guidance. We considered these classrooms as communities in which the members share some basic social rules, meanings and values (see, for example, Tharp *et al.*, 2000). Characteristic of a classroom community is that members do not only share goals or assignments, but basic cognitive values, tools and working strategies are also taken as shared.

In our research project we were curious to find out whether the intensive engagement of pupils in schematising activities would result in the acknowledgement of

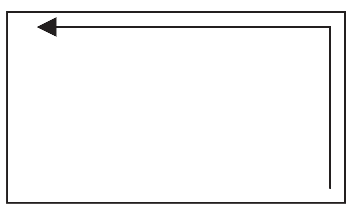

Figure 5. Question 3 in schematising

schematising as a collective activity, shared as a group's strategy for addressing problems. By means of video-analysis, we were able to acquire a deeper understanding of the quality of schematising activities and to show that schematising can be a collective activity in the classroom community. It was expected that schematising in the experimental condition was of more quality and would become an integral part of the classroom community—as compared with the control condition.

Mathematical achievement (Test Moment 3). Eight months after the intervention period was ended, we administered a National standardised CITO test to determine the learning outcomes of children in both groups. CITO is a Dutch norm-referenced standardised test used in primary education. The CITO test intends to measure children's mathematical abilities by comparing results with the average test score in the Netherlands. The CITO test was conducted in February, 2004 (post-test).

Sampling, data collection and analysis

The schematising test. To analyse the 'schematising test', a list of criteria was developed by the researchers to score the answers on the test. The test consisted of three questions and children could obtain either zero, one or two points for each question. Two points were given for correct answers; 1 point was awarded to questions that showed some part of the answer; while a zero was awarded if a child demonstrated no understanding of the question. The maximum score was six points if each of the three questions was answered correctly. Once these data were collected, the mean scores of each school were compared. These scores were then analysed using the *independent samples t-test.* The Levene's test for equality of variances was also performed. Because we expected our experimental group to perform better than our control group, we tested for one-tailed-significance.

The 'schematising test' is actually an instrument that is best evaluated through observation. Therefore, a second observer was asked to analyse the videos in order to establish observer agreement with regard to this test. Thus we could determine the Cohen's Kappa, which was 0.93, meaning that there was sufficient agreement for the test to be trustworthy.

After the data was collected, it became evident that there was a considerable amount of missing information. Therefore, only the collected data from this test were included in the analyses (N=54).

Collectivity: the schematisation score

For this analysis, we created an observation instrument and worked by means of event-sampling and a rating scale. The frequency of the video recording sessions and classroom visits by the teacher-trainer was the same for both groups. Lessons that were expected or likely to become schematising activities were videotaped from beginning to end during the experimental year of our research programme for both conditions. In these lessons, children participated in schematising activities or in activities that the teacher expected to result in schematising. We made use of event-sampling methods as we tallied the activities. By means of a rating scale we scored each activity on quality and collectivity.

The number of schematising activities differed per school. Although the same number of lessons were videotaped in both conditions, the experimental condition showed a lot more schematising activities. Since we were interested in the quality of schematising and in the extent to which schematising was integrated in the classroom culture, we analysed each activity separately and we made a score that expressed how intensively (frequency, quality and collectivity) schematising occurred within that activity. On the basis of literature search we constructed a list of characteristics that was used for the analysis of each of the activities (see Appendix). The total score on this list was added up per school. By dividing this total score by the number of activities on that school we calculated the average value of schematising in that school. We take this average as an expression of the schematisation score in the school involved. The mean scores of each school and each condition are calculated and finally compared by use of the *independent samples t-test*.

In order to be sure that the analyses we had to do and the conclusions we had to draw were reliable, we determined the interobserver agreement by asking another researcher to analyse the schematising activities. This researcher analysed 33% percent of the activities. Cohen's Kappa of the total instrument was 0.94, which means the agreement is satisfactory and the instrument is taken to be reliable.

Results

Can schematising be learned?

Table 3 displays the results of the 'schematising test'. An overview of the total experimental group compared with the total control group is demonstrated in the table.

Based on the data presented in Table 3, we can conclude that there is a significant difference between the total experimental group and the total control group with respect to the results on the schematising test and the mean scores of the groups. The effect size is .59, which is a large effect (see Cohen, 1988) in favour of the experimental group. We can conclude that 1) schematising can be learned and 2) at the start of grade three the experimental group positively differs from the control group.

Table 3. The results of the experimental group (N=35) compared to the results of the control group (N=19) on the schematising test

Condition	Mean	SD	df	F	t	p
Experimental	3.43	1.481	52	1.054	4.558	0.00
Control	1.74	1.195	44.258			

Effects of schematising on learning processes

Table 4 displays the results of the degree of 'quality' and 'collectivity' in schematising in both conditions. The total score is what we refer to as 'schematisation score'.

Table 4 shows that if we compare the experimental group with the control group, we can conclude that the experimental schools have a mean score on schematisation of 32.26 points and the control schools have a mean score of 17.67 points. This is a difference of almost 15 points, which is a 54 per cent higher score. Moreover, the difference on the total mean score is significant, as well as all the differences on the two specific categories.

Effects of schematising on learning outcomes

Children's elementary understanding of numbers was tested by administering the UGT test at the start of our research project (Test Moment 1). According to Van Luit *et al.* (1998), this is a prerequisite for mathematical development.

Table 5 indicates that, on the UGT test, the control group had a higher score than the experimental group. In a one-way analysis of variance, a significant difference of 5.58% on the pre-test between the two groups was found ($F = 5.7868$, $p = .018$).

Test Moment 3 was conducted in February of 2004, during the post-intervention period, which took place when the children were in grade three. This is the first year that children are given formal mathematics education and thus is also the first year in which children are confronted with mathematical tasks. The precise mean scores are presented in Table 6.

In order to determine whether the difference between the groups was significant, a variance-covariance analysis was conducted. The dependent variable was Test Moment 3, with Test Moment 1 as a covariate. A significant difference ($p = .000$) in favour of the experimental condition was found. The effect size was .29. In educational environments, effect sizes between +.20 and –.25 are considered meaningful (Slavin, 1996).

From the results presented above, we can conclude that the experimental group had significantly better scores on the CITO test in the post intervention period when compared with the control group. On the basis of this finding, we can claim that our experiment did indeed have a positive effect on children's learning outcomes in math-

Table 4. Experimental versus control

Subject	Condition	Mean	SD	df	F	*t*	*p*
Quality	Control	12.78	3.53	30.00	4.623	5.013	0.000
	Experimental	23.87	6.22				
Collectivity	Control	4.89	3.95	13.77	0.023	2.298	0.019
	Experimental	8.39	3.68				
Total score (schematisation score)	Control	17.67	6.58	19.13	1.376	5.157	0.000
	Experimental	32.26	8.58				

Table 5. Scores on the pre-test (UGT) for the experimental group (N=75) and the control group (N=58)

	Mean score	SD	Min	Max
Control programme				
Pre-test	52.90	12.443	21	75
Experimental programme				
Pre-test	47.32	13.813	16	75

ematics. The children in our experimental group were already prepared for participation in mathematical tasks since they had previous experience with dynamic schematising. These children were already familiar with mathematical reasoning and therefore had fewer difficulties with the mathematical tasks and tests in grade three. The children in the control group were not systematically exposed to schematising in early childhood and the results show that these children had more difficulty with mathematical tasks. If there was even schematising in the control group, it was always schematising of a static form.

Correlations and regression analyses

In order to conduct a regression analysis, we use in this section the original performance scores on CITO. In the foregoing sections the scores were expressed as standardised percentage scores according to the national CITO guidelines. These percentage scores make a (visual) presentation of the descriptives easier to comprehend. However, for statistical reasons we prefer the original scores for the correlations and regressions.

To give a first impression about the bivariate relations between the main variables in the regression analysis, we have presented the correlations in Table 7.

We then choose to consider the effects of the variables ‘UGT, 2002’ and ‘condition’ on the learning outcomes generated in February, 2004. A multiple linear regression analysis was conducted, in which a dummy variable was created for ‘condition’ (0 stands for the control group and 1 for the experimental group). The variables ‘UGT September, 2002’ (pre-test) and ‘condition’ were subsequently

Table 6. Learning outcomes (in percentages) on the CITO test for the experimental group (N=75) and the control group (N=58)

	Mean score	SD	Min	Max
Control programme				
Test moment 3 (CITO February, 2004)	50.06	17.89	7.79	84.42
Experimental programme				
Test moment 3 (CITO February, 2004)	55.26	19.45	19.48	100

Table 7. Correlations between the variables to be included in the regression analysis

	UGT September 2002	CITO February 2004	Condition
UGT September, 2002	1	.694	−.205
CITO February, 2004	.694	1	.136
Condition	−.205	.136	1

included in the equation. No interactions were found. The outcomes are presented in Table 8.

By referring to Table 8, we can conclude that the variable 'UGT September, 2002' explained 48.2% of the variance for the third moment of testing (CITO February, 2004 in the post-intervention period). Additionally, the variable 'condition' explained another 7% over and above the variance already explained by the previous variables. Thus, in this study we were able to explain 55% of the post-test variance.

Conclusion and discussion

With regard to the 'schematising test', we found that the pupils in the experimental group had a mean score that was almost two points higher than the mean score of the pupils in the control group. This difference is significant and relevant. This outcome cannot be a result of 'spontaneous' cognitive development, as the performances of the control group on the schematising test were considerably poorer. We therefore conclude that there is a significant difference between the way the experimental participants dealt with schematising and the way the control participants did.

The results on the 'schematisation score' display that schematising is more part of the classroom culture in the experimental classroom. In the control condition there were only a few activities to analyse, because many of the activities the children participated in could not be labelled as schematising activities. In the experimental condition, however, almost all the activities generated schematising activities. According to the research results it can be concluded that the schematising activities in the experimental condition are part of the classroom culture. In the comparison between the control and the experimental condition a significant difference was found in mean score on schematisation. The differences in scores on each sub-category of the schematisation

Table 8. Summary of Multiple Linear Regression Analysis for variables predicting the scores on the CITO test administered in February, 2004 (post-test)

Variable	R^2	R^2 change	F change	SigFch	B	SE B	β
UGT September, 2002 (pre-test)	.482	.482	91.971	.000	1.431	.134	.736
Condition	.550	.068	14.802	.000	7.811	2.030	.264

'test' were also striking. The mean score of the experimental condition was significantly higher than the score of the control condition.

The results of the CITO test administered in February, 2004, in the post-intervention period, demonstrated a significant difference in the learning outcomes in favour of the experimental group when compared with the control group. It is important to note that no further explicit training on schematising was given when the children were in grade three. At this stage, both the control group and the experimental group received a similar mathematics education.

Overall, we conclude that our hypothesis was confirmed. Pupils in the experimental group outperformed their counterparts in the control group on schematising, on quality and collectivity in schematising and on the CITO test. Learning how to deal with schematisations and learning to participate in schematising activities that are interesting and meaningful for young children in early childhood education gives these pupils an advantage over others and thus results in better performance in later development. A gap in comprehension seems to be bridged by means of our experiment.

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Appendix

The scoring-list for quality

Criteria	Explanation	Examples	Score	Category
Nature of the schematisation				
Static	A representative schematisation	There is a resemblance between schema and object like a portrait	1	S
		A drawing of what the child is looking at: like a map/building	1	S
	An iconic schematisation	Represent every object you see	1	S
	An idiosyncratic schematisation	A meaningless drawing (at least for someone who did not make it)	1	S
	A pictographic schematisation	A representation of shape, colour, place, shadow	1	S
Dynamic	A process	Schematisation of development	3	S
	Action/movement	Schematisation of narratives, songs, melodies, noises, maps *with* routes to follow	3	S
	Change	Change is pointed out between what happened first and last	3	S
	Relations	Cause-effect	3	S
	Reasoning/ Thought line	First this, second that …	3	S
Form of the eventual schematisations	Drawing		1	S
	Diagram		2	
	Graph		2	
	Map	Like a construction plan of a building	2	
	Narrative		1	
	Table		1	
	List		1	
	Symbols	Linking words Linking letters Linking lines Using words to make clear what is drawn Numerals,+, –, =, x, etc.	1	
Capacities and interests as a starting point	Is the activity meaningful for everybody?		1	C
	Is everybody interested in this activity?	Is it a stimulating activity (the process of schematising)	1	C

Appendix *(continued)*

Criteria	Explanation	Examples	Score	Category
	Teachers do justice to children's capacities, interests, etc.	An activity starts because of children's own need (the start of the activity)	2	C
	Teacher stimulates further thinking and children's ideas	Designing new symbols and schemes	1	C
Improve mathematical thinking: enrich play activities by:	Tasks invented by the teacher	Such as solving sums	1	I
	An activity invented by teacher **and** students	Several ideas are integrated into one	1	I
	Emerging from earlier activities	Like making a design as a result of a building		I
		- static	1	
		- dynamic	3	
	Teacher creates a meaningful context	Stimulating new activities and schematisations	1	I
	Representing	Represent children's thoughts and ideas symbolically	1	I
	Students practise with **own** notation systems		2	I
Are Activities appropriate for schematisation	An activity appropriate for schematisation	Static: a drawing of what you have been building	1	A
		Dynamic: a drawing of what happened in a story	3	
	Is schematisation of the activity meaningful	Is it meaningful to make a design of a building	3	A
	Introducing another way of thinking: introduction of tools to regulate an activity or stimulate schematising	For example: a notating system to notate the activity:		A
		static	1	
		dynamic	3	

Appendix *(continued)*

Criteria	Explanation	Examples	Score	Category
Reflection on the schematising activity or schematisation	Teacher makes clear what the sub-goals are		1	R
	Teacher revoices	Regulate children's language	1	R
	By discussion		1	R
	By giving feedback		1	R
	By reflection on the relation between sign and meaning	'Is this what you meant to do?'	3	R
	By working on difficulties	In translation of representations of concrete into written or drawn or the the other way around	1	R
		Making clear the value of symbols and math	1	
	Children reflect on own work		1	R

The scoring-list for collectivity

Criteria	Explanation	Examples	Score	Subscale
Collectivity	Teacher acts as member		1	CC
	Asking questions (teacher or children)		1	CC
	Children introduce rules		1	CC
	Children follow a model		1	CC
	Teacher stimulates children's interaction in order to share meanings(with regard to schematising activities)	Ask children to discuss about the activity or plans	3	CC
	Control each other's work		1	CC
	Solve each other's conflicts		1	CC
	Stimulation of each other's involvement	Ask stimulating questions	1	CC
	Negotiation of meaning	Are meanings being shared? Is there discussion about several ways of notating or schematising	3	CC
	Specific rules of notation are followed by everybody	Like an arrow to point out direction or movement	2	CC
	By reminding each other of …	Rules, agreements …	1	CC
	A schematising activity emerging from schematisation showing that the schematisation was understood	Reading maps	1	I
		Reading building designs or designing an own plan emerging from an own building	1	
	Enrichment of the(play) activity through schematisations; this is a base of further development	By the teacher	1	I
		By the children	3	I
	Adding an object to the activity to work out its meaning	By the teacher	1	I
		By the children	3	
	Children introduce own tools		1	I

Overview

Sub-category	Subscale	Designation	Maximum Score
Quality		Q	**37**
	Nature of the schematisation	S	**5**
	Activity based on children's own capacities and interests	C	**5**
	Improve mathematical thinking and enrich activities	I	**9**
	Are activities appropriate for schematisation?	A	**9**
	Reflection on the schematising activity	R	**9**
Collectivity		CC	**26**
Total score			**63**